

UNIVERSITY OF SARGODHA, SARGODHA

NOTIFICATION

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20/11/12

No.UOS/Acad/7499

Dated: 20.11.2012

On the recommendation of Academic Council, the Vice-Chancellor has, in exercise of the powers vested in him under section 13(3) of University of Sargodha Ordinance 2002, on behalf of the Syndicate, been pleased to approve the revised syllabus of M.Sc Mathematics (annual System) for implementation w.e.f 2012-14 session. Copy of approved syllabus is annexed herewith.

**Note:-** Revised syllabus of M.Sc Mathematics (annual System) can also be downloaded from university website - www.uos.edu.pk.

*(Ch. FAROOQ AHMAD)*  
(Ch. FAROOQ AHMAD)  
Assistant Registrar (Acad)  
for Registrar

**Distribution:-**

1. The Chairperson  
Department of Mathematics
2. Controller of Examinations
3. All Principals of Affiliated Colleges
4. Web Developer (for uploading on university website)
5. Notification file

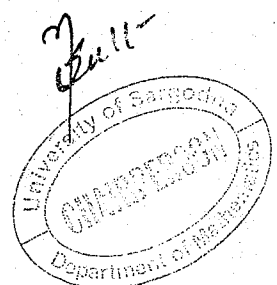
**C.C:**

- Secretary to the Vice-Chancellor
- P.A. to Registrar

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# Course outlines

M. Sc. Mathematics  
(Annual System)



### M.Sc Mathematics (Two Years Program)

**Eligibility:** The candidate having "Mathematics A Course and B Course" or "Applied Mathematics and Pure Mathematics" or equivalent in their B.A/B.Sc.

**Structure:** All courses of Part-I are compulsory. However, in Part-II three courses are compulsory and the students will be required to choose three more courses from the list of optional courses.

**Paper Pattern:** The paper pattern will be as following.

- i) 40% will comprise of objective.
- ii) 60% will be subjective.

Objective questions will be compulsory.

Regulation for M.Sc Mathematics Students:

- i) There shall be a total of 1200 marks for M.Sc (Mathematics) degree.
- ii) There shall be five papers in Part-I and six papers in Part-II. Each paper shall carry 100 marks.
- iii) There shall be a Viva Voce Examination at the end of M.Sc. Part-II carrying 100 marks.

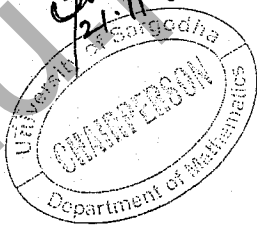
The Viva Voce Examination shall be conducted from the compulsory papers of Part-I.

M.Sc Part-I		
Sr. #	Course Title	Marks
1	Paper I: Real Analysis	100
2	Paper II: Algebra (Group Theory and Linear Algebra)	100
3	Paper III: Complex Analysis and Differential Geometry	100
4	Paper IV: Mechanics	100
5	Paper V: Topology & Functional Analysis	100
<b>Total Marks</b>		<b>500</b>
M.Sc Part-II		
Sr. #	Course Title	Marks
1	Paper VI: Advanced Analysis	100
2	Paper VII: Methods of Mathematical Physics	100
3	Paper VIII: Numerical Analysis	100
4	Optional Paper*	100
5	Optional Paper*	100
6	Optional Paper*	100
7	Viva Voce	100
<b>Total Marks</b>		<b>700</b>
<b>Grand Total</b>		<b>1200</b>

\* These three optional papers have to be chosen from the list of optional papers (M.Sc. Part-II).



List of Optional Papers (M.Sc. Part-II)		
Sr. #	Course Title	Marks
1	Paper IX: Mathematical Statistics	100
2	Paper X: Computer Applications	100 (50+50)
3	Paper XI: Group Theory	100
4	Paper XII: Rings and Modules	100
5	Paper XIII: Number Theory	100
6	Paper XIV: Fluid Mechanics	100
7	Paper XV: Special Relativity and Analytical Dynamics	100
8	Paper XVI: Theory of Approximation and Splines	100



**Course outline**  
**M.Sc Mathematics (Annual System)**  
**(for Private Candidate)**

**APPENDIX**  
**(Detailed Outlines of Courses of Study)**

**M.Sc. Part-I Papers**

**Paper I: Real Analysis**

Five questions to be attempted, selecting at least two questions from each section.

**Section I (5/9)**

*The Real Number System*

Ordered sets. Fields, The field of real, The extended real number system, Euclidean spaces.

*Numerical Sequences and Series*

Convergent sequences, Subsequences, Cauchy sequences, upper and lower limits, Series, Series of non-negative terms, the number, the root and ratio tests, power series.

*Continuity* The Limit of a function, Continuous functions, Continuity and compactness, Continuity and connectedness, Discontinuities.

*Differentiation*

The derivative of a real function, mean-value theorems, the continuity of derivatives. *Real-Valued Functions of Several Variables* Partial derivatives and differentiability, derivatives and differentials of composite functions. Change in the order of partial derivation, implicit functions, inverse functions, Jacobians, Maxima and minima (with and without side Conditions).

**Section II (4/9)**

*The Riemann-Stieltjes Integrals*

Definition and existence of the integral properties of the integral, integration and differentiation, functions of bounded variation.

*Sequences and Series of Functions*

Uniform convergence, uniform convergence and continuity, uniform convergence and integration, uniform convergence.

*Improper integrals*

Tests for convergence of improper integrals, infinite series and infinite integrals, Beta and Gamma functions and their properties.

***Books Recommended***

1. W. Rudin, Principles of Mathematical Analysis, McGraw-Hill 1976.
2. T. M. Apostol, Mathematical Analysis, Addison-Wesley, 1974.
3. W. Kaplan, Advanced Calculus, Addison-Wesley, 2002.

**Paper II: Algebra (Group Theory and Linear Algebra)**

Five questions to be attempted, selecting at least two questions from each section.

**Section I (4/9)**

*Group Theory*

Cyclic groups, coset decomposition of a group, Lagrange's theorem and its consequences, conjugacy classes, centralisers and normalisers, normal subgroups, homomorphisms of groups. Cayley's theorem, Quotient groups, fundamental theorem of homomorphism, isomorphism theorems, endomorphisms and automorphisms of groups, Direct product of groups, Characteristic and fully invariant subgroups, simple groups (Definition and examples). Double cosets, Sylow theorems.

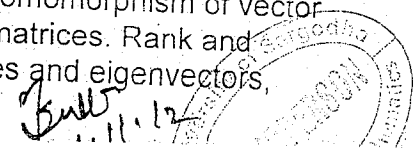
**Section II (5/9)**

*Ring Theory (2/9)*

Definition and example of rings, special classes of rings, Fields, Ideals, Ring homomorphisms, Quotient rings, prime and maximal ideals. Field of quotients.

*Linear Algebra (3/9)*

Vector spaces. Subspaces. Bases. Dimension of a vector space. Homomorphism of vector spaces, Quotient spaces, Dual spaces. Linear transformation and matrices. Rank and nullity of a linear transformation, characteristic equation, eigenvalues and eigenvectors.



similar matrices, diagonalization of matrices. Orthogonal matrices and orthogonal transformations.

### Books Recommended

1. J.J. Rottman, The Theory of Groups: An Introduction, Allyn & Bacon, Boston, 1984.
2. J. Rose, A Course on Group Theory, C.U.P. 1978.
3. I.N. Herstein, Topics in Algebra, Xerox Publishing Company, 1975.
4. G. Birkhoff and S. MacLane, A Survey of Modern Algebra, Macmillan, New York, 1998.
5. I. Macdonald, The Theory of Groups, Oxford University Press, 1968.
6. P.M. Cohn, Basic Algebra, Vol. I, London: John Wiley, 2002.
7. D. Burton, Abstract and Linear Algebra, Addison-Wesley publishing Co, 1972.
8. P.B. Battacharya, S.K. Jain and S.R. Nagpaul, Basic Abstract Algebra, C.U.P., 1994.
9. N. Jacobson, Basic Algebra- I, Vol. II Freeman, 2009.

### Paper III: Complex Analysis and Differential Geometry

Five questions to be attempted selecting not more than two questions from each section.

#### Section I (3/9)

##### *The concept of analytic functions*

The complex number, points sets in the complex plane, functions of a complex variable, General properties of analytic rational functions. The  $n$ th power, polynomials rational functions, linear transformations. Basic properties of linear transformations, mapping for problems, stereographic projections, Mapping by rational functions of second order, The exponential and the logarithmic functions, the trigonometric functions, infinite series with complex terms, Power series, infinite products.

#### Section II (3/9)

##### *Integration in the complex domain*

Cauchy's theorem, Cauchy's integral formula and its applications, Laurent's expansion, isolated singularities of analytic functions, mapping by rational functions, the residue theorem and its applications, the residue theorem, definite integrals, partial fraction, expansion of  $\cot 2z$ , the arguments principle and its application.

##### *Analytic continuation*

The principle of analytic continuation, the monodromy theorem, the inverse of a rational function, the reflection principle.

#### Section III (3/9)

##### *Differential Geometry*

Space curves, arc length, tangent, normal and binormal, curvature and torsion of a curve. Tangent surface, involutes and evolute. Existence theorem for a space curve. Helices, Curves on surfaces, surfaces of revolution, Helicoids. Families of curves, Developables, Developables associated with space curves. Developables associated with curves on surfaces, the second fundamental form. Principal curvatures, lines of curvature.

### Book Recommended

1. W. Kaplan, Introduction to Analytic Functions, Addison-Wesley, 1966.
2. L.L. Pennissi, Introduction to Complex Variables, Holt Rinehart, 1976.
3. R.V. Churchill, Complex Variables and Applications, J.W. & Brown & Roger F. Verthey, 5th Edition, 1995.
4. J.E. Mersden, Basic Complex Analysis, W.H. Freeman & Co., San Francisco, 1973.
5. T.J. Wilmore, An Introduction to Differential Geometry, Oxford Calarendon Press, 1966.
6. D. Laugwitz, Differential and Riemannian Geometry, Academic Press, New York, 1965.
7. C.E. Weatherburn, Differential Geometry, Cambridge University Press, 1961.

### Paper IV: Mechanics

Five questions to be attempted, selecting at least two questions from each section.

#### Section I: Vector and Tensor Analysis (4/9)

##### *A. Vector Calculus (2/9)*

Gradient, divergence and curl of point functions, expansion formulas, curvilinear coordinates, line, surface and volume integrals, Gauss's, Green's and Stokes's theorems.

##### *B. Cartesian Tensors (2/9)*

Tensors of different ranks, Inner and outer products, contraction theorem, Kronecker tensor and Levi-Civita tensor, Applications to Vector Analysis.

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## Section II (5/9)

### *Mechanics*

General Motion of a rigid body, Euler's theorem and Chasles' theorem, Euler's angles, Moments and products of inertia, inertia tensor, principal axes and principal moments of inertia, Kinetic energy and angular momentum of a rigid body. Momental ellipsoid and equiproportional systems, Euler's dynamical equations and their solution in special cases. Heavy symmetrical top, equilibrium of a rigid body, General conditions of equilibrium, and deduction of conditions in special cases.

### **Books Recommended**

1. F. Chorlton, A Text Book of Dynamics, CBS Publishers, 2004.
2. H. Jeffery, Cartesian Tensors, Cambridge University Press.
3. F. Chorlton, Vector and Tensor Methods, Ellis Horwood Publisher, Chichester, UK, 1977.
4. Lunn, M., Classical Mechanics (Oxford).
5. Griffine, Theory of Classical Dynamics, C.U.P.

### **Paper V: Topology & Functional Analysis**

Five questions to be attempted, selecting at least two questions from each section.

#### **Section I (4/9) (Topology)**

Definition, Open and closed sets, subspaces, neighbourhoods, limit points, closure of a set, comparison of different topologies, bases and sub-bases, first and second axiom of countability, separability, continuous functions and homeomorphisms, weak topologies, Finite product spaces. Separation axioms ( $T_0$ ,  $T_1$ ,  $T_2$ ), regular spaces, completely regular spaces, normal spaces, compact spaces, connected spaces.

#### **Section II (5/9) (Functional Analysis)**

### *Metric Spaces*

Definition & examples, Open and closed sets, Convergences, Cauchy sequence and examples, completeness of a metric space, completeness proofs.

### *Banach spaces*

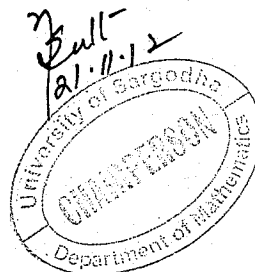
Normed linear spaces, Banach spaces, Quotient spaces, continuous and bounded linear operators, linear functional, linear operator and functional on finite dimensional spaces.

### *Hilbert spaces*

Inner product spaces, Hilbert spaces (definition and examples), Orthogonal complements, Orthonormal sets & sequences, conjugate spaces, representation of linear functional on Hilbert space, reflexive spaces.

### **Books Recommended**

1. G.F. Simon, Introduction to Topology and Modern Analysis, McGraw Hill Book Company, New York, 2003.
2. J. Willard, General Topology, Addison-Wesley Publishing Company, London.
3. E. Kreyszig, Introduction to Functional Analysis with Applications, John Wiley and Sons, 1989.
4. W. Rudin, Functional Analysis, McGraw Hill Book Company, New York, 1991.
5. N. Dunford and J. Schwartz, Linear Operators (Part-I General Theory), Inter science Publishers, New York, 1988.



## M.Sc. Part II Papers

### **Paper VI: Advanced Analysis**

Five questions to be attempted, selecting at least one question from each section.

#### **Section I (2/9)**

##### *Advanced Set Theory*

Equivalent sets, Countable and uncountable sets, The concept of cardinal number, addition and multiplication of cardinals, Cartesian products as sets of functions, addition and multiplication of ordinals, partially ordered sets, axiom of choice, statement of Zorn's lemma.

#### **Section II (5/9)**

##### *Lebesgue Measure*

Introduction, outer measure, Measurable sets and Lebesgue measure, A non-measurable set, Measurable functions, the Lebesgue Integral and the Riemann integral, the Lebesgue integral of a bounded function over a set of finite measure. The integral of a non-negative function. The general Lebesgue integral. Convergence in measure.

#### **Section III (2/9)**

##### *Ordinary Differential Equations*

Hypergeometric function  $F(a,b,c,l)$  and its evaluation. Solution in series of Bessel differential equation. Expression for  $J_n(x)$  when  $n$  is half odd integer, recurrence formulas for  $J_n(x)$ . Series solution of Legendre differential equation. Rodrigues formula for polynomial  $P_n(x)$ . Generating function for  $P_n(x)$ , recurrence relations and the orthogonality of  $P_n(x)$  functions.

##### **Books Recommended**

1. A.A. Fraenkel, Abstract Set Theory, North-Holland Publishing, Amsterdam, 1966.
2. Patrick Suppes, Axiomatic Set Theory, Dover Publications, Inc., New York, 1972.
3. P.R. Halmos, Naive Set Theory, New York, Van Nostrand, 1960.
4. B. Rotman & G.T. Kneebone, The Theory of Sets and Transfinite Numbers, oldbourne, London.
5. P.R. Halmos, Measure Theory, Von Nostrand, New York, 1950.
6. W. Rudin, Real and Complex Analysis, McGraw Hill, New York, 1966.
7. R.G. Bartle, Theory of Integration.
8. H.L. Royden, Real Analysis, Prentice-Hall, 1997.
9. E.D. Rainville, Special Functions, Macmillan and Co.
10. N.N. Lebedev, Special Functions and their Applications, Prentice-Hall.

### **Paper VII: Methods of Mathematical Physics**

At least one question to be selected from each section, five questions in all.

#### **Section I**

##### *Partial Differential Equations of Mathematical Physics (2/9)*

Formation and classification of partial differential equations. Methods of separation of variables for solving elliptic, parabolic and hyperbolic equations. Eigenfunction expansions.

#### **Section II**

##### *Sturm-Liouville System and Green's Functions (2/9)*

Some properties of Sturm-Liouville equations. Sturm-Liouville systems. Regular, periodic and singular Sturm-Liouville systems. Properties of Sturm-Liouville Systems. Green's function method. Green's function in one and two dimensions.

##### *Integral Equations (1/9)*

Formulation and classification of integral equations. Degenerate Kernels, Method of successive approximations.

#### **Section III**

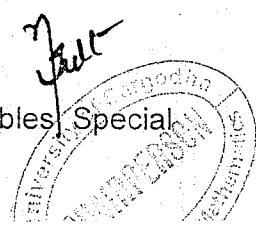
##### *Integral Transforms and their Applications (2/9)*

Definition and properties of Laplace transforms. Inversion and convolution theorems. Application of Laplace transforms to differential equations. Definition and properties of Fourier transforms. Fourier integrals and convolution theorem. Applications to boundary value problems.

#### **Section IV**

##### *Variational Methods (2/9)*

Euler-Lagrange equations when integrand involves one, two, three and  $n$  variables, Special cases of Euler-Lagrange equations. Necessary conditions for existence of an extremum of a functional, constrained maxima and minima.





**Books Recommended**

1. E.L. Butkov, Mathematical Physics, Addison-Wesley, 1973.
2. H. Sagan, Boundary and Eigenvalue Problems in Mathematical Physics.
3. R.P. Kanwal, Linear Integral Equations, Academic Press, 1971.
4. Tyn Myint-U: & L. Denbnath, Partial Differential Equations, Elsevier Science Pub. 1987.
5. G. Arfken, Mathematical Methods for Physics, Academic Press, 1985.
6. I. Stakgold, Boundary Value Problems of Mathematical Physics, Vol. II, Macmillan, 1968.

**Paper VIII: Numerical Analysis**

Five questions to be attempted, selecting at the most two questions from each section.

**Section I (3/9)***Linear and Non-Linear Equations*

Numerical methods for nonlinear equations. Regula-falsi method. Newton's method. Iterative method. Rate and conditions of convergence for iterative and Newton's methods. Gaussian elimination method. Triangular decomposition (Cholesky) method and its various forms. Jacobi, Gauss-Seidel and iterative methods for solving system of linear equations. Ill-conditioned system and condition number. Error estimates and convergence criteria for system of linear equations. Power and Raleigh method for finding eigenvalues and eigenvectors.

**Section II (3/9)***Interpolation and Integration*

Various methods including Aitkins and Lagrange interpolation, error estimate formulae for interpolation and its applications, Numerical differentiation, trapezoidal, Simpson and quadrature formulae for evaluating integrals with error estimates.

**Section III (3/9)***Difference and Differential Equations*

Formulation of difference equations, solution of linear (homogeneous and inhomogeneous) difference equations with constant coefficients, the Euler and the modified Euler method. Runge-Kutta methods and predictor-corrector type methods for solving initial value problems along with convergence and instability criteria. Finite difference, collocation and variational method for boundary value problems.

**Books Recommended**

1. C. Gerald, Applied Numerical Analysis, Addison-Wesley Publishing Company, 1978.
2. A. Balfour & W.T. Beveridge, Basic Numerical Analysis with Fortran, Heinemann Educational Books Ltd., 1977.
3. Shan and Kuo, Computer Applications of Numerical Methods, Addison-Wesley, National Book Foundation, Islamabad, 1972.

**OPTIONAL PAPERS****Paper IX: Mathematical Statistics**

Five questions to be attempted, selecting at least two questions from each section.

**Section I (4/9)***Probability*

The postulates of probability and some elementary theorems, addition and multiplication rules, Baye's rule, probability functions, probability distributions (discrete, uniform, Bernoulli, binomial, hypergeometric, geometric, negative binomial, Poisson). Probability densities, the uniform, exponential, gamma, beta and normal distributions, change of variable.

**Section II (5/9)***Mathematical Expectation*

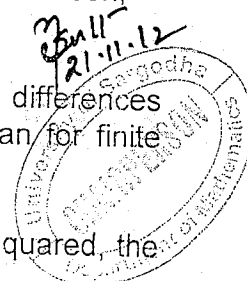
Moments, moment generating functions, moments of binomial, hypergeometric, Poisson, gamma, beta and normal distributions.

*Sums of Random Variables*

Convolutions, moment generating functions, the distribution of the mean, differences between means, differences between proportions, the distribution of the mean for finite populations.

*Sampling Distributions*

The distribution of  $\bar{x}$ , the chi-squared distribution and the distribution of s-squared, the F distribution, the t distribution.



*Regression and Correlation*

Linear regression, the methods of least squares, correlation analysis.

**Books Recommended**

1. J.E. Freund, *Mathematical Statistics*, Prentice-Hall Inc., 1992.
2. Hogg & Craig, *Introduction to Mathematical Statistics*, Collier Macmillan, 1958.
3. Mood, Greyill & Boes, *Introduction to the Theory of Statistics*, McGraw Hill.

**Paper X: Computer Applications**

The evaluation of this paper will consist of two parts:

1. Written examination: 50 marks
2. Practical examination: 50 marks

(The practical examination includes 10 marks for the notebook containing details of work done in the Computer Laboratory and 10 marks for the oral examination). It will involve writing and running programmes on computational projects. It will also include familiarity with the use of Mathematical Recipes subroutines (and MATHEMATICA in calculus and graphing of functions).

**Course Outline for the Written Examination**

Five questions to be attempted, selecting at least two questions from each section.

**Section I (4/9)***Computer Orientation*

General introduction to digital computers, their classes and working. Concepts of low-level and high-level computer languages, an algorithm and a programme. Problem-solving process using digital computers including use of flow-charts.

*Programming in FORTAN: (Fortran 90, 95)*

Arithmetic expressions, Assignment statements, I/O statements including the use of I, F, E, H and X specifications. Computed IF statements, computed Go To-statement, Logical expressions and logical IF-statements, Nested Do-loops, Do-WHILE loop. Subscripted variables and arrays, DIMENSION statements, Implied Do-loops, Data statement, COMMON and EQUIVALENT statements, SUBROUTINE subprogrammes and FUNCTION subprogrammes.

**Section II (5/9)***Computational projects in Fortran*

- a) Bisection method, Regula falsi method, Newton-Raphson method for solving non-linear equations.
- b) Gaussian elimination with different pivoting strategies, Jacobi and Gauss—Seidel iterative methods for systems of simultaneous linear equations.
- c) Trapezoidal rule, Simpson's rule and Gaussian method of numerical integration.
- d) Modified and improved Euler's methods; Predictor-Corrector methods for finding the numerical solution of IVP's involving ODE's.

Note: Practical examination will be of two hours duration in which one or more computational projects will be examined.

**Books Recommended**

1. M.L. Abell and J.P. Braselton, *Mathematica Handbook*, New Yourk, 1992.
2. T.J. Akai, *Applied Numerical Methods*, J. Wiley, 1994.
3. J.H. Mathews, *Numerical Methods for Computer Science, Engineering and Mathematics*, Prentice-Hall, 1987.

**Paper XI: Group Theory**

Five questions to be attempted, selecting at least two questions from each section.

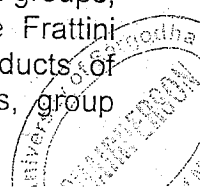
**Section I (5/9)**

Characteristic and fully invariant subgroups, normal products of groups, Holomorph of a group, permutation groups, cyclic permutations and orbits, the alternating group, generators of the symmetric and alternating groups, simplicity of  $A_n$ ,  $n > 5$ . The stabilizer subgroups.  $n$  = series in groups. Zassenhaus lemma, normal series and their refinements, composition series, principal or chief series.

**Section II (4/9)**

Solvable groups, definition and examples, theorems on solvable groups. Nilpotent groups, characterisation of finite nilpotent groups, upper and lower central series, the Frattini subgroups, free groups, basic theorems, definition and examples of free products of groups, linear groups, types of linear groups. Representation of linear groups, group algebras and representation modules.

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**Books Recommended**

1. I.D. Macdonald, The Theory of Groups, Oxford, Clarendon Press, 1975.
2. H. Marshall, The Theory of Groups, Macmillan, 1967.
3. David S. Dummit and Richard M. Foote, Abstract Algebra, 3rd Ed. John Wiley and Sons, 2004.
4. Michiel Hazewinkel, N. Mikhailovna, V. Kirichenko, Algebras, Rings and Modules, Kluwer Academic Publishers, 2004.

**Paper XII: Rings and Modules**

Five questions to be attempted, selecting at least two questions from each section.

**Section I (5/9)***Rings*

Construction of new rings, Direct sums, polynomial rings, Matrix rings. Divisors, Units and associates, Unique factorization domains. Principal ideal domains and Euclidean domains. Field Extensions. Algebraic and transcendental elements. Degree of extension. Algebraic extensions. Reducible and Irreducible polynomials. Roots of polynomials.

**Section II (4/9)***Modules*

Definition and examples, submodules, Homomorphisms and quotient modules. Direct Sums of modules. Finitely generated modules, Torsion Modules, Free modules. Basis, rank and endomorphisms of free modules. Matrices over rings and their connection with the basis of a free module. A module as the direct sum of a free and a torsion module.

**Books Recommended**

1. I.N. Herstein, Topics in Algebra, Xerox Publishing Company Mass, 1972.
2. B. Hartley & T.O. Hauvkes, Rings, Modules and Linear Algebra, Chapman and Hall Ltd., London, 1970.
3. R.B.J.T. Allenly, Rings, Fields and Groups, An Introduction to Abstract Algebra, Edward Arnold, 1985.
4. Michiel Hazewinkel, N. Mikhailovna, V. Kirichenko, Algebras, Rings and Modules, Kluwer Academic Publishers, 2004.

**Paper XIII: Number Theory**

Five questions to be attempted, selecting at least two questions from each section.

**Section I (5/9)***Analytic Number Theory*

*Congruences* Elementary properties, Residue classes and Euler's function. Linear congruences and congruences of higher degree. Congruences with prime moduli. The theorems of Fermat, Euler and Wilson.

*Primitive roots and indices*

Integers belonging to a given exponent, composite moduli, Indices.

*Quadratic Residues*

Composite moduli, Legendre symbol, Law of quadratic reciprocity, the Jacobi symbol.

*Number-Theoretic Functions*

Mobius function, the function  $[x]$ , the symbols  $O$ ,  $O$  and  $-$  and their basic properties.

*Diophantine Equations*

Equations and Fermat's conjecture for  $n=2$ ,  $n=4$ .

**Section II (4/9)***Algebraic Number Theory*

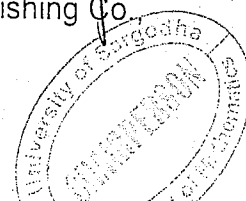
Algebraic numbers and integers, Units and Primes in  $R[v]$  ideals. Arithmetic of ideals congruences. The norm of an ideal. Prime ideals. Units of algebraic number field.

*Applications to Rational Number Theory*

Equivalence and class number. Cyclotomic field  $K_p$  Fermat's equation. Kummer's theorem, the equation,  $x^2 + 2 - y^3$ , pure cubic fields. Distribution of primes and Riemann's zeta function, the prime number theorem.

**Book Recommended**

W.J. Leveque, Topics in Number Theory, Vols. I and II, Addison-Wesley Publishing Co, 2002.



**Paper XIV: Fluid Mechanics**

Five questions to be attempted, selecting at most two questions from each section.

**Section I (3/9)***Introduction*

Fluid and continuity hypotheses; Surface and Body forces, Stress at a point, Viscosity, Newton's viscosity law; Viscous and inviscid flows, Laminar and Turbulent flows, Compressible and Incompressible flows; Lagrangian and Eulerian descriptions; Local; Connective and total rates of change; Conservation of mass.

**Section II (3/9)***Inviscid Fluids*

Irrotational motions, Boundary conditions, streamlines, vortex lines and vortex sheets, Kelvin's minimum energy theorem, Conservation of Linear momentum, Bernoulli's theorem and its applications, Circulations, Rate of change of circulation (Kelvin's theorem), axially symmetric motion, Stokes's stream function, Two-dimensional motion, Stream function, Complex potential and some potential flows, Sources, sinks and doublets, Circle theorem, Method of images, Blasius theorem, aerofoil and the theorem of Kutta and Joukowski, vortex motion, Karman's vortex street.

**Section III (3/9)***Viscous Fluids*

Constitutive equations, Navier-Stokes's equations, exact solutions of Navier-Stokes's equations: Steady unidirectional flow, Poiseuille flow, Couette flow, unsteady unidirectional flow, sudden motion of a plane boundary in a fluid at rest, Flow due to an oscillatory boundary. Equations of motion relative to a rotating system, Ekman flow, Dynamical similarity and the Reynold's number, Boundary layer concept and its governing equations, Flow over a flat plat (Blasius solution); Reynold's equations of turbulent motion.

**Books Recommended**

1. H. Schlichting, Boundary-Layer Theory, McGraw Hill.
2. Y. Chia-Shun, Fluid Mechanics, McGraw Hill, 1974.
3. I.L. Distworth, Fluid Mechanics, McGraw Hill.
4. I.G. Curie, Fundamentals of Mechanics of Fluids, McGraw Hill, 1974.
5. R.W. Fox & A.T. McDonald, Introduction to Fluid Mechanics, John Wiley & sons.

**Paper XV: Special Relativity and Analytical Dynamics**

Note: Five questions to be attempted, selecting at least one question from section I.

**Section I (3/9)***Special Relativity*

Fundamental concepts, The Lorentz transformation, Time dilation and Lorentz-Fitzgerald contraction, Transformation of velocities, four-velocity and four-acceleration. Relativistic dynamics, relativistic equations of motion, relativistic mass, linear momentum, four-force, relativistic kinetic energy, four momentum.

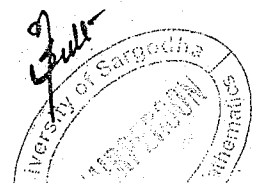
**Section II (6/9)***Analytical Dynamics*

- a) Generalized coordinates, Holonomic and non-holonomic systems. D' Alembert's principle, d-delta rule.
- b) Lagrange's Theory of Holonomic Systems.
  - (i) Equations of Lagrange, Generalization of Lagrange equations, Quasi-coordinates and Lagrange equations in quasi-coordinates. (ii) First integrals of Lagrange equations of motion, Energy integral and Whittaker's equations, ignorable coordinates and Routhian function, Noether's theorem.
- c) Lagrange's Theory of Non-Holonomic Systems.
 

Equations of Lagrange for non-holonomic systems with and without Lagrange multipliers, Chaplygin's equations.
- d) Hamilton's Theory.
  - (i) Hamilton's principle, Generalized momenta and phase space, Hamilton's equations. (ii) Canonical transformations, Generating functions, the Lagrange and Poisson brackets, Bilinear covariants, infinitesimal exact transformations. (iii) Hamilton-Jacobi theorem.

**Books Recommended**

1. M. Saleem and M. Rafique, Special Relativity, Ellis Horwood, 1992.
2. Rosser, Special Relativity, 1972.
3. W. Rindler, Introduction to Special Relativity, Oxford, 1982.



4. D.T. Greenwood, Classical Dynamics, Prentice-Hall, Inc., 1965.
5. H. Goldstein, Classical Mechanics, Addison-Wesley, 1964.
6. L.A. Pars, Treatise of Analytical Dynamics, Heninemann Press, London, 1965.

**Paper XVI: Theory of Approximation and Splines**

Five questions to be attempted, selecting at least two questions from each section.

**Section I: (4/9)**

*Euclidean Geometry*

Basic concepts of Euclidean Geometry, Scalar and Vector functions, Barycentric Coordinates, Convex Hull, Matrices of Affine Maps: Translation, Rotation, Scaling, Reflection and Shear.

*Approximation using Polynomials*

Linear Interpolation, Least squares polynomial curve fitting, Lagrange's Method, Hermite's Methods, Divided Differences Methods.

**Section II (5/9)**

*Parametric Curves (Scalar and Vector Case)*

Algebraic Form. Hermite Form. Control Point Form, Bernstein Bezier Form, Matrix Forms of Parametric Curves, Algorithms to compute B.B. Form, Convexhull Property, Affine invariance property, variation diminishing property, Rational Quadratic Form, Rational Cubic Form.

*Spline Functions*

Splines, Cubic Splines, End Conditions of Cubic Splines: Clamped conditions, Natural conditions,  $2^{nd}$  Derivative conditions, Periodic conditions, Not a knot conditions, General Splines: Natural Splines, Periodic Splines, Truncated Power Function, Representation of spline in terms of truncated power functions, Odd degree interpolating splines.

**Books Recommended**

1. Gerald Farin, Curves and Surfaces for Computer Aided Geometric Design A Practical Guide, Academic Press. Inc..
2. I.D. Faux, Computational Geometry for Design and Manufacture, Ellis Horwood.
3. Richard H. Bartels. An Introduction to Spline for use in Computer Graphics and Geometric Modeling, Morgan Kaufmann Publisher, Inc.
4. Carl de Boor, A Practical Guide to Splines, Springer Verlag 1978.
5. Schumaker, Spline Functions: Basic Theory, John Wiley 1981.

